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"SIMPLE" SOLUTIONS OF THE EQUATIONS OF DYNAMICS FOR A POLYTROPIC GAS

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The notion of a "simple" solution of a system of differential equations that admit a local Lie group G of transformations of the basic space is considered as an invariant H-solution of type $(0,0)$ with respect to the subgroup $H \subset G$. Such solutions are attractive since they are described by explicit formulas that provide a clear physical interpretation for them. For gas-dynamic equations with a polytropic gas law, all simple solutions that are not related to special forms of gas flow are listed. Examples of simple solutions are given and the collapse phenomenon, which has been previously studied for barochronic flows, is described.

Introduction. The equations of gas dynamics (EGD) of a *polytropic gas* for the velocity vector $\boldsymbol{u} = (u, v, w)$, density ρ, pressure p, and entropy S as functions of time t and the coordinates $\boldsymbol{x} = (x, y, z)$ have the form

$$\rho D\boldsymbol{u} + \nabla p = 0, \quad D\rho + \rho \operatorname{div} \boldsymbol{u} = 0, \quad Dp + \gamma p \operatorname{div} \boldsymbol{u} = 0, \quad p = S\rho^{\gamma} \qquad (D = \partial_t + \boldsymbol{u} \cdot \nabla) \tag{1}$$

with the adiabatic exponent $\gamma = \text{const} > 0$. It is known [1] that system (1) has fairly wide symmetry, namely, *it admits* (following Lie) the 13-dimensional group G_{13} of transformations of the space $R^9(t, \boldsymbol{x}, \boldsymbol{u}, \rho, p)$.

The Lie algebra of the operators L_{13} acting on R^9 that corresponds to the group G_{13} has the form

$$\begin{gathered} X_i = \partial_{x^i}, \quad X_{i+3} = t\partial_{x^i} + \partial_{u^i}, \quad X_{i+6} = \varepsilon^l_{ik}(x^k \partial_{x^l} + u^k \partial_{u^l}) \quad (i = 1, 2, 3), \\ X_{10} = \partial_t, \quad X_{11} = t\partial_t + x^i \partial_{x^i}, \quad X_{13} = t\partial_t - u^i \partial_{u^i} + 2\rho\partial_\rho, \quad X_{14} = \rho\partial_\rho + p\partial_p \end{gathered} \tag{2}$$

in the designations $(x, y, z) = (x^1, x^2, x^3)$, $(u, v, w) = (u^1, u^2, u^3)$, $\partial_{x^i} = \partial/\partial x^i$, $\partial_t = \partial/\partial t$, etc., ε^l_{ik} is a standard skew-symmetric tensor, and $\varepsilon^3_{12} = 1$. The repeated indexes denote summation. (Number 12 is reserved for the operator X_{12}, which extends L_{13} to L_{14} in the case $\gamma = 5/3$.)

The purpose of the SUBMODEL program is to find all the possibilities induced by the indicated symmetry property for constructing classes of exact solutions of the EGD [2]. Such possibilities are provided for by the *subgroups* $H \subset G_{13}$ (the subalgebras $L \subset L_{13}$), each of which can generate a class of solutions described in terms of the *invariants* of the group H. The invariants are related by differential equations that follow from (1) and, generally speaking, have reduced dimension. These equations form a *factor system* that describes the *H-submodel* for the EGD.

The H-submodels are classified by their *types* (σ, δ), where σ is the *rank* of the submodel (the number of invariant independent variables) and δ is its *defect* (the number of noninvariant unknown functions). *Invariant* submodels are distinguished by the condition $\delta = 0$.

Submodels of type (0,0) hold a special position among all H-submodels. For them, the unknown invariants are constants and the factor system is reduced to a system of finite relations between these constants.

The solutions of the EGD obtained from submodels of type (0,0) are called *simple solutions*.

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The simplest member of the set of simple solutions is a *constant* solution (a quiescent gas with constant density and pressure). On the whole, this set (infinite) is rich in nonequivalent classes of various simple solutions.

The present work is concerned with preliminary classification of simple solutions of the EGD (1). Classes of simple solutions possessing a particular generality are selected, their singularities are indicated, and particular examples are given.

Selection of Subalgebras. Submodels of type (0,0) are generated by the four-dimensional subalgebras $L_4 \subset L_{13}$. All these subalgebras with accuracy up to similarity are listed in the optimal system of the subalgebras ΘL_{13} [3], which contains a total of 290 representatives of $L_{4,i}$ (with ordinal numbers $i = 1, 2, \ldots, 290$). However, not all of them are suitable for constructing submodels of type (0,0). First of all, the subalgebra $L_{4,i}$ should obey the following two necessary conditions.

I. $L_{4,i}$ does not have invariants of the form $j = j(t, \boldsymbol{x})$.

II. The factor system obtained is consistent.

Thus, only 124 representatives satisfy condition I, of which 39 lead to inconsistent factor systems, and, hence, 85 representatives satisfy both conditions I and II. The submodels of type (0,0) generated by them have been constructed, but, at the same time, many of them provide a description of particular cases of special gas flows. At the present stage of investigation, it is not expedient to consider simple solutions for the gas flow types III–V indicated below and the corresponding systems of equations.

III. *Barochronic flows*, defined by the relation $p = p(t)$:

$$D\boldsymbol{u} = 0, \quad \operatorname{div} \boldsymbol{u} = -p'/\gamma p.$$

They include isobaric flows with $p = \text{const}$.

IV. *Thermal flows*, characterized by constant density $\rho = \text{const} \neq 0$:

$$D\boldsymbol{u} + \nabla p = 0, \quad Dp = 0, \quad \operatorname{div} \boldsymbol{u} = 0.$$

V. *Isothermal flows*, characterized by constant velocity of sound $c = \text{const}$; for $\gamma = 1$,

$$D\boldsymbol{u} + \nabla e = 0, \quad De + \operatorname{div} \boldsymbol{u} = 0 \qquad (e = \ln \rho),$$

and for $\gamma \neq 1$, system IV is the case.

These models should be given an independent group analysis, which has not yet been completed. It is pertinent to note that system IV has not yet been reduced to involution. The barochronic model III was studied in [4].

Results. Besides the subalgebras generated by submodels III–V for gas flows, there are 34 representatives of $L_{4,i}$ indicated in Tables 1 and 2. Their additional classification by the dimensions of the solutions obtained is compiled here.

The *dimension* of a simple solution is the number N of independent variables of the form $\lambda_i = \lambda_i(t, \boldsymbol{x})$ ($i = 1, \ldots, N$) on which the solution obtained depends. The number N is given by the formula

$$N = \text{g.r.} \frac{\partial(u, v, w, \rho, p)}{\partial(t, x, y, z)},$$

where g.r. denotes the "general rank" of the indicated Jacobian matrix. A solution of dimension N is denoted by the symbol DN. In all, there are 2 solutions $D1$, 14 solutions $D2$, 10 solutions $D3$, and 8 solutions $D4$.

Table 1 lists 26 subalgebras $L_{4,i}$ that generate simple solutions of class DN for $N = 1, 2, 3$. The dimension N is given in the first column, the number i of the subalgebra $L_{4,i}$ from the optimal system ΘL_{13} [3] is given in the second column, the basis of the operators of this subalgebra in the third columns, the limitations of parameters that guarantee that condition I is satisfied and the solution falls in the corresponding class DN are given in the fourth column, and the independent variables on which the solution depends are given in the fifth column. The basic operators are written in abbreviated form, only with the numbers of the generating operators (2). For example, the expression (2)11 − 13 + a14 denotes the operator $2X_{11} - X_{13} + aX_{14}$, where

TABLE 1

Subalgebras $L_{4,i}$ Generating Simple Solutions of Classes DN ($N = 1, 2, 3$)

N	i	Basis $L_{4,i}$	Limitations	Characteristic variables
1	73	2, 3, 11, $7 + a13 + b14$	$a \neq 0$	x/t
	75	2, 3, 11, $13 + a14$		x/t
2	62	1, $a4 + 6, 11, 13 + b14$		$y/t, z/t$
	87	2, 3, $4 + a6 + 10, (2)11 - 13 + b14$		$t, x - t^2/2$
	91	2, 3, $4 + 10, (2)11 - 13 + a14$		$t, x - t^2/2$
	93	2, 3, $4 + 10, 7 - 2a11 + a13 + b14$		$t, x - t^2/2$
	231	1, 11, $7 + a13, b13 + 14$	$b \neq 0$	$r/t, \theta$
	232	1, 11, $13, 7 + a14$	$a \neq 0$	$r/t, \theta$
	234	1, 10, $7 + a11 + b13, c11 + d13 + 14$	$c \neq 0$	r, θ
	235	1, 10, $a11 + 13, 7 + b11 + c14$	$a \neq 0$	r, θ
	239	1, 10, $2 + 13$, $a2 + b3 + 14$	$b \neq 0$	t, x
	276	2, 3, $7 + a10 - b11 + b13$, $c10 - d11 + d13 + 14$	$bc - ad \neq 0$	t, x
	277	2, 3, $7 + a11 + b13, c11 + d13 + 14$	$bc - ad \neq 0$	$x/t, x$
	283	2, 3, $a10 - 11 + 13, 7 + b10 + 14$	$b \neq 0$	t, x
	284	2, 3, $a10 - 11 + 13, b10 + 14$	$b \neq 0$	t, x
	285	2, 3, $a11 + 13, 7 + b11 + c14$	$b \neq 0$	$x/t, x$
	286	2, 3, $a11 + 13, b11 + 14$	$b \neq 0$	$x/t, x$
3	57	5, 6, $7 + a13 + b14, 11$	$a \neq 0$	$x/t, y/t, z/t$
	59	5, 6, $11, 13 + a14$		$x/t, y/t, z/t$
	217	1, $7 + a11, b4 + 13, c11 + 14$	$c \neq 0$	t, r, θ
	228	4, 11, $7 + a13, b11 + 14$	$b \neq 0$	$x/t, r/t, \theta$
	229	4, 11, 13, $7 + a14$		$x/t, r/t, \theta$
	241	1, $4 + 10$, $7 + 2a11 - a13$, $2b11 - b13 + 14$	$b \neq 0$	t, r, θ
	242	1, $4 + 10$, $(2)11 - 13, 7 + a11$		t, r, θ
	260	1, $a4 + 6, b11 + 13, c11 + 14$	$c \neq 0$	$t, y/t, z/t$
	264	1, $2 + 4, a10 - 11 + 13, b10 + 14$		t, y, z

TABLE 2

Subalgebras $L_{4,i}$ Generating Simple Solutions of Classes D4

Subclass	i	Basis $L_{4,i}$	Limitations
$D4_1$	216	4, $7 + a11, b11 + 13, c11 + 14$	$c \neq 0$
$D4_2$	252	5, 6, $7 + a11 + b13, c11 + d13 + 14$	$ad - bc \neq 0$
	255	5, 6, $a11 + 13, 7 + b11 + c14$	$b \neq 0$
	256	5, 6, $a11 + 13, b11 + 14$	$b \neq 0$
$D4_3$	248	5, 6, $a1 + 2 + 13, b1 + d2 + c3 + 14$	$b \neq 0$
	249	5, 6, $a1 + 13, b1 + 7 + c14$	$b \neq 0$
	251	5, 6, $a1 + 13, b1 + 14$	$b \neq 0$
	254	5, 6, $a1 + 7 + b13, c1 + d13 + 14$	$ad - bc \neq 0$

a is an arbitrary parameter. The polar coordinates (r, θ) in the plane (y, z) are introduced by the formulas

$$y = r\cos\theta, \quad z = r\sin\theta. \tag{3}$$

Table 2 lists the remaining eight subalgebras $L_{4,i}$ that generate simple solutions of the class $D4$. These solutions can be divided into three subclasses $D4_k$ ($k = 1, 2, 3$), in each of which the solution formulas have the same form. Therefore, here the notation of the subclass is indicated in the first column, the second and third columns are the same as in Table 1, and the limitations of parameters are given in the fourth column. This provides a compact description of all simple solutions of the class $D4$.

Simple Solutions $D4_1$. The solution is represented as

$$u = \frac{1}{t}(x + Ur), \quad v = \frac{1}{t}(Vy - Wz), \quad w = \frac{1}{t}(Wy + Vz), \quad \rho = t^{A-1}r^{B-2}e^{K\theta}R, \quad p = t^{A-3}r^{B}e^{K\theta}P,$$

where (r, θ) are polar coordinates (3). Here U, V, W, R, and P are in essence the invariants of the subalgebra, and the constants A, B, and K are expressed in terms of the parameters (a, b, c) of the series of subalgebras, namely: $A = 3 - b/c$, $B = (b+1)/c$, and $K = -a/c$.

Substitution of the representation into the EGD (1) leads to the factor system

$$UV = 0,\ K\frac{P}{R} = (1-2V)W,\ B\frac{P}{R} = W^2 - V^2 + V,\ BV + KW + A = 0,\ 2\gamma V = 3 - \gamma.$$

The solution must satisfy the conditions $V^2 + W^2 \neq 0$ and $B^2 + K^2 \neq 0$, the first of which guarantees that the solution falls in the class $D4$, and the second prevents the solution from falling in the class of barochronic solutions. The following possibilities arise:

1^0. $V(V-1)(2V-1) \neq 0$. By virtue of the expression $V = (3-\gamma)/2\gamma$, this is equivalent to the inequality $(\gamma - 1)(2\gamma - 3)(\gamma - 3) \neq 0$. Here $U = 0$, and the constants B, K, and P/R are expressed in terms of A, V, and W by the formulas

$$B = \frac{W^2 - V^2 + V}{(V-1)(V^2+W^2)}A,\quad K = \frac{(1-2V)W}{(V-1)(V^2+W^2)}A,\quad \frac{P}{R} = \frac{1}{A}(V-1)(V^2+W^2), \tag{4}$$

where the condition of positiveness of the value of P/R must be satisfied, namely, $(V-1)A > 0$, which is equivalent to the condition $(\gamma - 1)A < 0$. The solution depends on four arbitrary constants V, W, A, and γ, related by the indicated formulas.

2^0. $V = 0$. A solution exists only for $\gamma = 3$. Formulas (4) are brought to the form

$$B = -A,\quad KW = -A,\quad P/R = -W^2/A \qquad (W \neq 0).$$

We obtain the simple solution

$$u = \frac{x}{t} + U\frac{r}{t},\quad v = -W\frac{z}{t},\quad w = W\frac{y}{t},\quad \rho = t^{A-1}r^{-A-2}e^{-A\theta/W}R,\quad p = t^{A-3}r^{-A}e^{-A\theta/W}P,$$

and the condition $A < 0$ must be satisfied. The solution depends on three arbitrary constants U, W, and A.

3^0. $V = 1/2$ A solution exists only for $\gamma = 3/2$. Here $U = 0$, and formulas (4) are brought to the form

$$B = -2A,\quad K = 0,\quad P/R = -(W^2 + 1/4)/2A.$$

We obtain the simple solution

$$u = \frac{x}{t},\quad v = \frac{y - 2Wz}{2t},\quad w = \frac{2Wy + z}{2t},\quad \rho = t^{A-1}r^{-2A-2}R,\quad p = t^{A-3}r^{-2A}P,$$

and the condition $A < 0$ must be satisfied. The solution depends on two arbitrary constants W and A.

4^0. $V = 1$. A solution exists only for $\gamma = 1$. Here $U = 0$, and from the factor system it follows that $W \neq 0$ and $K \neq 0$. We obtain the relations $A = 0$ and $B = -KW$, which give the simple solution

$$u = \frac{x}{t},\quad v = \frac{y - Wz}{t},\quad w = \frac{Wy + z}{t},\quad \rho = t^{-1}r^{-KW-2}e^{K\theta}R,\quad p = t^{-3}r^{-KW}e^{K\theta}P,$$

where $P/R = -W/K$ for $KW < 0$. The solution depends on two arbitrary constants W and K.

Simple Solutions $D4_2$. The solution is represented as

$$u = U\frac{x}{t},\quad v = \frac{y}{t},\quad w = \frac{z}{t},\quad \rho = t^{A+2}x^{B-2}R,\quad p = t^{A}x^{B}P.$$

Here U, R, and P are in essence invariants of any subalgebra of the subclass $D4_2$ (Table 2), and the constants A and B are uniquely expressed in terms of the corresponding parameters a, b, c, and d of these subalgebras. These constants must satisfy the inequalities $A + B \neq 0$ and $B \neq 0$, the first of which ensures that the solution falls in the class $D4$, and the second prevents the solution from falling in the class of barochronic solutions.

Substitution of the representation into the EGD (1) leads to the factor system

$$B\frac{P}{R} = U - U^2,\quad (B-1)U + A + 4 = 0,\quad (B+\gamma)U + A + 2\gamma = 0.$$

The condition $U - U^2 \neq 0$ is necessary, which is equivalent to $(\gamma-1)(\gamma-2) \neq 0$. Here the constant $B \neq 0$ remains arbitrary, and the remaining constants are expressed in terms of it:

$$U = 2\frac{2-\gamma}{\gamma+1}, \qquad A = 2\frac{\gamma-2}{\gamma+1}B - \frac{6\gamma}{\gamma+1}, \qquad \frac{P}{R} = -\frac{6}{B}\frac{(\gamma-1)(\gamma-2)}{(\gamma+1)^2}.$$

Here B and γ must satisfy the inequalities $B(\gamma-1) \neq 2\gamma$ and $B(\gamma-1)(\gamma-2) < 0$. When these inequalities are fulfilled, the solution depends on two arbitrary constants B and γ.

Simple Solutions $D4_3$. The solution is represented as

$$u = \frac{1}{t}, \quad v = \frac{1}{t}(y + f(\lambda)), \quad w = \frac{1}{t}(z + g(\lambda)), \quad \rho = t^{-2}e^{A\lambda}R, \quad p = t^{-4}e^{A\lambda}P, \quad \lambda = x - \ln t.$$

Here f and g are, generally speaking, arbitrary functions of the variable λ. For particular submodels from Table 2, they have the following values:

- $f = m\lambda$ and $g = n\lambda$ for $i = 248$,
- $f = g = 0$ for $i = 249$ and 251,
- $f = h\cos(m\lambda)$ and $g = h\sin(m\lambda)$ for $i = 254$,

and the constants m, n, h, and A are uniquely expressed in terms of the parameters a, b, c, and d of the corresponding subalgebras.

The condition that the solution falls in the class $D4$ is $A \neq 0$. Substitution of the representation into the EGD gives a factor system that is reduced to the relations $\gamma = 2$ and $AP = R$, and, hence, $A > 0$ must hold here. The solution depends on a maximum of three arbitrary constants, for example, m, n, and A.

Collapse. The phenomenon of collapse, namely, reversal of density into infinity at $t = 0$, is typical of gas flows described by simple solutions of the class $D4$. In contrast to barochronic gas flows [4], the geometry of collapse in this case is more complex since the trajectories of gas particles are not rectilinear.

Below, for greater physical obviousness, in the simple solutions considered we perform the transformation "reversal of time" $t \to 1 - t$ as $\boldsymbol{u} \to -\boldsymbol{u}$, which is admitted by the EGD. The gas flow is considered in the interval $0 \leqslant t \leqslant 1$. Then, at $t = 0$ there is no singularity, and collapse occurs at $t = 1$.

The particle trajectories are described by solutions of the system

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{u}(\boldsymbol{x}, t), \quad \boldsymbol{x}\Big|_{t=0} = \boldsymbol{x}_0. \tag{5}$$

Let a solution of this system be $\boldsymbol{x} = \boldsymbol{X}(t, \boldsymbol{x}_0)$. Elimination of time t gives the equations of the projection $L(\boldsymbol{x}_0)$ of the particle trajectory issuing from the point $\boldsymbol{x}_0$ at $t = 0$ onto the space $R^3(\boldsymbol{x})$. The points of the collapse are on these projections and can be obtained from the expressions $\boldsymbol{x}_1 = \boldsymbol{X}(1, \boldsymbol{x}_0)$. Since the Jacobian matrix $\partial \boldsymbol{X}/\partial \boldsymbol{x}_0$, generally speaking, degenerates at $t = 1$, the set of points of the collapse $M(\boldsymbol{x}_1)$ has a dimension less than three in $R^3(\boldsymbol{x})$, i.e., this can be a point, a line, or a two-dimensional surface (sometimes, located at infinity). Below, the collapse phenomenon is illustrated for two simple solutions of the class $D4$.

Solution $D4_1 2^0$. In cylindrical coordinates, the velocity field has the form [r and θ are determined in (3)]

$$u = -\frac{1}{1-t}(x + Ur), \quad v_r = 0, \quad v_\theta = \frac{r}{1-t}W,$$

where u, v_r, and v_θ are, respectively, the axial (on the x axis), radial, and circumferential components of the velocity vector. Then, the solution of the corresponding system (5) is

$$x = (x_0 + Ur_0)(1-t) - Ur_0, \quad r = r_0, \quad \theta = \theta_0 + W\ln(1-t).$$

The projections $L(\boldsymbol{x}_0)$ of the trajectories onto $R^3(\boldsymbol{x})$ are given by the equations

$$r = r_0, \quad x = (x_0 + Ur_0)\exp\frac{\theta - \theta_0}{W} - Ur_0.$$

The points of the collapse ($t = 1$) are defined by the equations $x_1 = -Ur_0$ and $r_1 = r_0$, and the coordinate θ_1 remains uncertain since the angle $\theta \to \infty$ as $t \to 1$ (if $W < 0$). The set of points of the collapse $M(\boldsymbol{x}_1)$ is

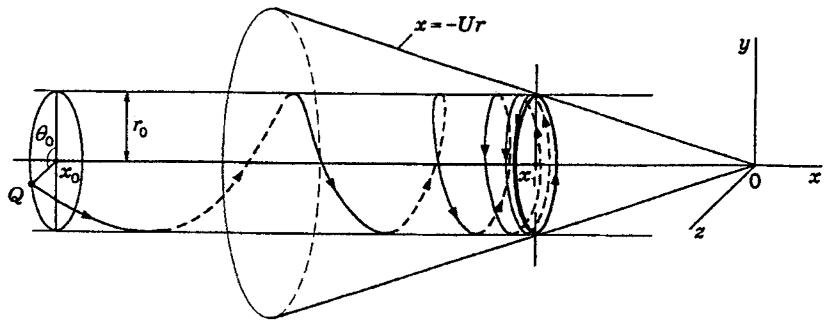

Fig. 1. Typical trajectory of a particle Q starting at $t = 0$ from the point (x_0, r_0, θ_0).

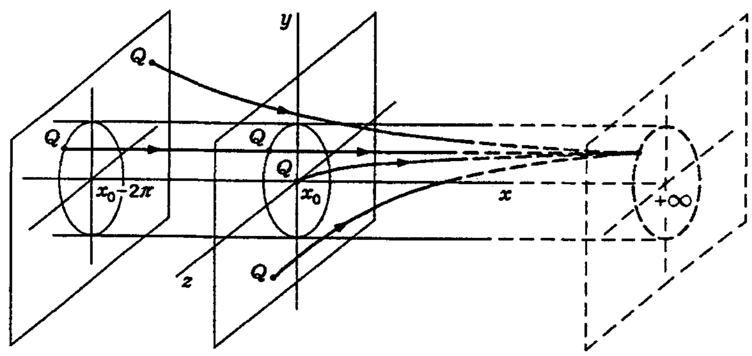

Fig. 2. Typical trajectories of particles Q starting at $t = 0$ from the planes $x = x_0$ and $x = x_0 - 2\pi$.

the cone described by the equation $x_1 = -Ur_1$. Any trajectory that issues at $t = 0$ from the point (x_0, r_0, θ_0) lies on a cylinder of radius r_0 (with the x axis) and is wound without restriction on it with an exponentially decreasing step, approaching the circle of the collapse — the section of the cylinder cut by the plane $x = -Ur_0$ [which coincides with the intersection of the cylinder and the cone $M(\boldsymbol{x}_1)$]. A typical trajectory is qualitatively shown in Fig. 1.

Solution $D4_3$ ($i = 254$). For simplicity, we assume that $m = 1$. Then, the velocity field can be written as

$$u = \frac{1}{1-t}, \quad v = -\frac{1}{1-t}(y + h\cos\lambda),$$
$$w = -\frac{1}{1-t}(z + h\sin\lambda), \quad \lambda = x + \ln(1-t),$$

and in the representation of the density and pressure, A must be replaced by $-A$.

The solution of the corresponding system (5) has the form

$$x = x_0 - \ln(1-t), \quad y = (y_0 + h\cos x_0)(1-t) - h\cos x_0,$$
$$z = (z_0 + h\sin x_0)(1-t) - h\sin x_0.$$

The projections $L(\boldsymbol{x}_0)$ of the trajectories onto $R^3(\boldsymbol{x})$ are given by the equations

$$y + h\cos x_0 = (y_0 + h\cos x_0)\mathrm{e}^{x_0 - x}, \quad z + h\sin x_0 = (z_0 + h\sin x_0)\mathrm{e}^{x_0 - x}$$

and are planar curves. The points of the collapse ($t = 1$) are given by the equations

$$x_1 = +\infty, \quad y_1 = -h\cos x_0, \quad z_1 = -h\sin x_0.$$

The set of points of the collapse $M(\boldsymbol{x}_1)$ is a circle of radius $|h|$ with center on the x axis at $x = +\infty$. The particle trajectories issuing at $t = 0$ from the planes $x = x_0$ and $x = x_0 + 2\pi n$ ($n = \pm 1, \pm 2, \ldots$) come at the same point on $M(\boldsymbol{x}_1)$. Typical trajectories are shown qualitatively in Fig. 2.

Conclusion. The above examples of simple solutions of the EGD (1) show that these solutions are far from being trivial: they describe rather complex gas flows. At the same time, the relative simplicity of the formulas of these solutions allows one to find the particle trajectories in finite form and to study the dynamics of the collapse in detail. The same circumstance can simplify the solution of a number of interesting gas-dynamic problems, such as continuation of the solution behind the collapse, conjugation of different simple solutions via weak and strong discontinuities, decay of an arbitrary discontinuity, etc. It is planned to publish a comprehensive list of all simple solutions of the EGD (1).

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